8.15 A cylindrical 1045 steel bar (Figure 8.34) is subjected to repeated compression-tension stress cycling along its axis. If the load amplitude is 22,000 N (4950 lb), compute the minimum allowable bar diameter to ensure that fatigue failure will not occur. Assume a factor of safety of 2.0.

Solution

From Figure 8.34, the fatigue limit stress amplitude for this alloy is 310 MPa (45,000 psi). Stress is defined in Equation 6.1 as \[ \sigma = \frac{F}{A_0} \]. For a cylindrical bar

\[ A_0 = \pi \left( \frac{d_0}{2} \right)^2 \]

Substitution for \( A_0 \) into the Equation 6.1 leads to

\[ \sigma = \frac{F}{A_0} = \frac{F}{\pi \left( \frac{d_0}{2} \right)^2} = \frac{4F}{\pi d_0^2} \]

We now solve for \( d_0 \), taking stress as the fatigue limit divided by the factor of safety. Thus

\[ d_0 = \sqrt[4]{ \frac{4F}{\pi \left( \frac{\sigma}{N} \right)} } \]

\[ = \sqrt[4]{ \frac{(4)(22,000 \text{ N})}{\pi \left( \frac{310 \times 10^6 \text{ N/m}^2}{2} \right)} } = 13.4 \times 10^{-3} \text{ m} = 13.4 \text{ mm} \ (0.53 \text{ in.}) \]
8.34 Steady-state creep rate data are given below for nickel at 1000°C (1273 K):

<table>
<thead>
<tr>
<th>$\dot{\varepsilon}_s$ (s⁻¹)</th>
<th>$\sigma$ [MPa (psi)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$</td>
<td>15 (2175)</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>4.5 (650)</td>
</tr>
</tbody>
</table>

If it is known that the activation energy for creep is 272,000 J/mol, compute the steady-state creep rate at a temperature of 850°C (1123 K) and a stress level of 25 MPa (3625 psi).

Solution

Taking natural logarithms of both sides of Equation 8.20 yields

$$\ln \dot{\varepsilon}_s = \ln K_2 + n \ln \sigma - \frac{Q_c}{RT}$$

With the given data there are two unknowns in this equation—namely $K_2$ and $n$. Using the data provided in the problem statement we can set up two independent equations as follows:

$$\ln \left(1 \times 10^{-4} \text{ s}^{-1}\right) = \ln K_2 + n \ln (15 \text{ MPa}) - \frac{272,000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1273 \text{ K})}$$

$$\ln \left(1 \times 10^{-6} \text{ s}^{-1}\right) = \ln K_2 + n \ln (4.5 \text{ MPa}) - \frac{272,000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1273 \text{ K})}$$

Now, solving simultaneously for $n$ and $K_2$ leads to $n = 3.825$ and $K_2 = 466 \text{ s}^{-1}$. Thus it is now possible to solve for $\dot{\varepsilon}_s$ at 25 MPa and 1123 K using Equation 8.20 as

$$\dot{\varepsilon}_s = K_2 \sigma^n \exp \left(\frac{-Q_c}{RT}\right)$$

$$= (466 \text{ s}^{-1})(25 \text{ MPa})^{3.825} \exp \left[-\frac{272,000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1123 \text{ K})}\right]$$

$$= 2.28 \times 10^{-5} \text{ s}^{-1}$$