Slip in Single Crystals

7.11 Sometimes \( \cos \phi \cos \lambda \) in Equation 7.2 is termed the Schmid factor. Determine the magnitude of the Schmid factor for an FCC single crystal oriented with its [100] direction parallel to the loading axis.

Solution

We are asked to compute the Schmid factor for an FCC crystal oriented with its [100] direction parallel to the loading axis. With this scheme, slip may occur on the (111) plane and in the [1\overline{1}0] direction as noted in the figure below.

The angle between the [100] and [1\overline{1}0] directions, \( \lambda \), may be determined using Equation 7.6

\[
\lambda = \cos^{-1} \left[ \frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{u_1^2 + v_1^2 + w_1^2} \sqrt{u_2^2 + v_2^2 + w_2^2}} \right]
\]

where (for [100]) \( u_1 = 1, v_1 = 0, w_1 = 0 \), and (for [1\overline{1}0]) \( u_2 = 1, v_2 = -1, w_2 = 0 \). Therefore, \( \lambda \) is equal to

\[
\lambda = \cos^{-1} \left[ \frac{1(1) + 0(-1) + 0(0)}{\sqrt{1^2 + 0^2 + 0^2} \sqrt{1^2 + (-1)^2 + 0^2}} \right]
\]

\[
= \cos^{-1} \left( \frac{1}{\sqrt{2}} \right) = 45^\circ
\]
Now, the angle $\phi$ is equal to the angle between the normal to the (111) plane (which is the [111] direction), and the [100] direction. Again from Equation 7.6, and for $u_1 = 1$, $v_1 = 1$, $w_1 = 1$, and $u_2 = 1$, $v_2 = 0$, and $w_2 = 0$, we have

\[
\phi = \cos^{-1}\left| \frac{(1)(1) + (1)(0) + (1)(0)}{\sqrt{(1)^2 + (1)^2 + (1)^2} \left[ (1)^2 + (0)^2 + (0)^2 \right]} \right|
\]

\[
= \cos^{-1}\left( \frac{1}{\sqrt{3}} \right) = 54.7^\circ
\]

Therefore, the Schmid factor is equal to

\[
\cos \lambda \cos \phi = \cos (45^\circ) \cos (54.7^\circ) = \left( \frac{1}{\sqrt{2}} \right) \left( \frac{1}{\sqrt{3}} \right) = 0.408
\]
7.12 Consider a metal single crystal oriented such that the normal to the slip plane and the slip direction are at angles of 43.1° and 47.9°, respectively, with the tensile axis. If the critical resolved shear stress is 20.7 MPa (3000 psi), will an applied stress of 45 MPa (6500 psi) cause the single crystal to yield? If not, what stress will be necessary?

Solution

This problem calls for us to determine whether or not a metal single crystal having a specific orientation and of given critical resolved shear stress will yield. We are given that $\phi = 43.1^\circ$, $\lambda = 47.9^\circ$, and that the values of the critical resolved shear stress and applied tensile stress are 20.7 MPa (3000 psi) and 45 MPa (6500 psi), respectively. From Equation 7.2

$$\tau_R = \sigma \cos \phi \cos \lambda = (45 \text{ MPa})(\cos 43.1^\circ)(\cos 47.9^\circ) = 22.0 \text{ MPa} \quad (3181 \text{ psi})$$

Since the resolved shear stress (22 MPa) is greater than the critical resolved shear stress (20.7 MPa), the single crystal will yield.
7.13 A single crystal of aluminum is oriented for a tensile test such that its slip plane normal makes an angle of 28.1° with the tensile axis. Three possible slip directions make angles of 62.4°, 72.0°, and 81.1° with the same tensile axis.

(a) Which of these three slip directions is most favored?

(b) If plastic deformation begins at a tensile stress of 1.95 MPa (280 psi), determine the critical resolved shear stress for aluminum.

Solution

We are asked to compute the critical resolved shear stress for Al. As stipulated in the problem, \( \phi = 28.1° \), while possible values for \( \lambda \) are 62.4°, 72.0°, and 81.1°.

(a) Slip will occur along that direction for which \( \cos \phi \cos \lambda \) is a maximum, or, in this case, for the largest \( \cos \lambda \). Cosines for the possible \( \lambda \) values are given below.

\[
\cos(62.4°) = 0.46 \\
\cos(72.0°) = 0.31 \\
\cos(81.1°) = 0.15
\]

Thus, the slip direction is at an angle of 62.4° with the tensile axis.

(b) From Equation 7.4, the critical resolved shear stress is just

\[
\tau_{crss} = \sigma_y \cos \phi \cos \lambda_{\text{max}}
\]

\[
= (1.95 \text{ MPa}) \left[ \cos (28.1°) \cos (62.4°) \right] = 0.80 \text{ MPa} \ (114 \text{ psi})
\]
7.30 A cylindrical specimen of cold-worked copper has a ductility (%EL) of 25%. If its cold-worked radius is 10 mm (0.40 in.), what was its radius before deformation?

Solution

This problem calls for us to calculate the precold-worked radius of a cylindrical specimen of copper that has a cold-worked ductility of 25%EL. From Figure 7.19c, copper that has a ductility of 25%EL will have experienced a deformation of about 11%CW. For a cylindrical specimen, Equation 7.8 becomes

$$\%CW = \left[ \frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \right] \times 100$$

Since $r_d = 10 \text{ mm (0.40 in.)}$, solving for $r_0$ yields

$$r_0 = \sqrt{\frac{r_d}{1 - \frac{\%CW}{100}}} = \sqrt{\frac{10 \text{ mm}}{1 - \frac{11.0}{100}}} = 10.6 \text{ mm (0.424 in.)}$$
7.D5 A cylindrical rod of 1040 steel originally 15.2 mm (0.60 in.) in diameter is to be cold worked by drawing; the circular cross section will be maintained during deformation. A cold-worked tensile strength in excess of 840 MPa (122,000 psi) and a ductility of at least 12%EL are desired. Furthermore, the final diameter must be 10 mm (0.40 in.). Explain how this may be accomplished.

Solution

First let us calculate the percent cold work and attendant tensile strength and ductility if the drawing is carried out without interruption. From Equation 7.8

\[
\%CW = \left( \frac{\pi \left( \frac{d_0}{2} \right)^2 - \pi \left( \frac{d_d}{2} \right)^2}{\pi \left( \frac{d_0}{2} \right)^2} \times 100 \right)
\]

\[
= \left( \frac{\pi \left( \frac{15.2 \text{ mm}}{2} \right)^2 - \pi \left( \frac{10 \text{ mm}}{2} \right)^2}{\pi \left( \frac{15.2 \text{ mm}}{2} \right)^2} \times 100 = 56\%CW \right)
\]

At 56%CW, the steel will have a tensile strength on the order of 920 MPa (133,000 psi) [Figure 7.19b], which is adequate; however, the ductility will be less than 10%EL [Figure 7.19c], which is insufficient.

Instead of performing the drawing in a single operation, let us initially draw some fraction of the total deformation, then anneal to recrystallize, and, finally, cold-work the material a second time in order to achieve the final diameter, tensile strength, and ductility.

Reference to Figure 7.19b indicates that 20%CW is necessary to yield a tensile strength of 840 MPa (122,000 psi). Similarly, a maximum of 21%CW is possible for 12%EL [Figure 7.19c]. The average of these extremes is 20.5%CW. Again using Equation 7.8, if the final diameter after the first drawing is \(d'_{0}\), then

\[
20.5\%CW = \left( \frac{\pi \left( \frac{d'_{0}}{2} \right)^2 - \pi \left( \frac{10 \text{ mm}}{2} \right)^2}{\pi \left( \frac{d'_{0}}{2} \right)^2} \times 100 \right)
\]

And, solving the above expression for \(d'_{0}\), yields
\[ d'_0 = \frac{10 \text{ mm}}{\sqrt{1 - \frac{20.5\%CW}{100}}} = 11.2 \text{ mm} \ (0.45 \text{ in.}) \]